Geometry Common Core Study Guide

Polygon Interior/Exterior Angles

Sum of interior angles = 180(n – 2)

Each interior angle (regular) = $\frac{180(n-2)}{n}$

Sum of exterior angles = 360°

Each exterior angle (regular) = $\frac{360}{n}$

Triangles

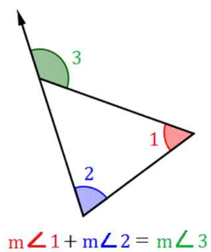
Classifying:

Sides: Scalene = no congruent sides
Isosceles = 2 congruent sides
Equilateral = 3 equal sides

Angles: Acute = all angles < 90°
Right = one right angle of 90°
Obtuse = one angle > 90°
Equiangular = 3 congruent angles (60°)

Sum of the angles of any triangle = 180°

Exterior angle = sum of 2 non-adjacent interior angles



Midsegment – segment joining the midpoints

- **Always parallel to the third side**
- **½ the length of the third side**
- **Splits the triangle into similar triangles**

Pythagorean Theorem

To find a missing side of a right triangle:

$$a^2 + b^2 = c^2$$

Where c is the hypotenuse

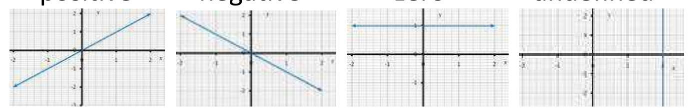
Supplementary Angles – sum to 180°
Complementary Angles – sum to 90°

Coordinate Geometry

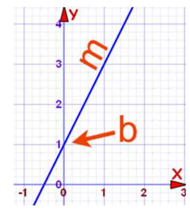
Linear $y = mx + b$ where m=slope & b=y-intercept

slope = $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

positive negative zero undefined



Collinear – on the same line
Parallel lines have the same slope
Perpendicular lines have negative reciprocal slopes (flip and change the sign)

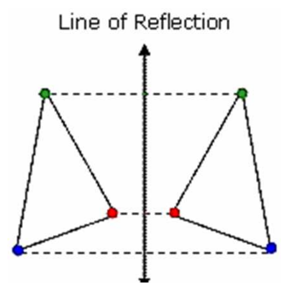


Midpoint = $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

Distance = $\sqrt{(x1-x2)^2 + (y1-y2)^2}$

Rigid Motion – transformations that preserve size and shape (Isometric)

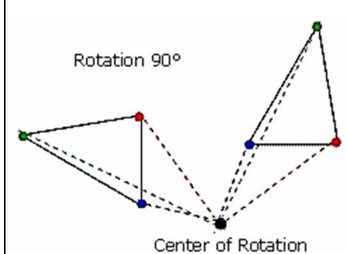
Reflection - FLIP



$r_{x\text{-axis}}(x, y) = (x, -y)$
$r_{y\text{-axis}}(x, y) = (-x, y)$
$r_{y=x}(x, y) = (y, x)$
$r_{y=-x}(x, y) = (-y, -x)$
$r_{origin}(x, y) = (-x, -y)$

Opposite isometry-orientation NOT preserved

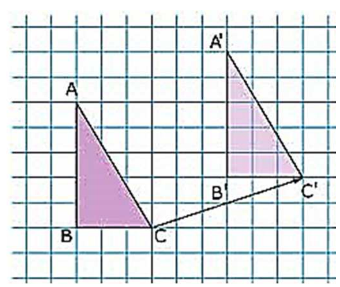
Rotation - TURN



$R_{90°}(x, y) = (-y, x)$
$R_{180°}(x, y) = (-x, -y)$
$R_{270°}(x, y) = (y, -x)$

Direct Isometry- preserves orientation

Translation – SLIDE



$T_{a,b}(x, y) = (x+a, y+b)$

Composition of Transformations

$R_{90°}$ o $T_{3,-4}$

←

Translation followed by a rotation

Factoring

Look for GCF first!!

GCF: $ab + ac = a(b + c)$

Difference Of Perfect Squares:
$x^2 - y^2 = (x + y)(x - y)$

Trinomial: $2x^2 + 7x - 15 =$
$(2x - 3)(x + 5)$

-3x
+10x
+7x

Similar Triangles Theorems

Similar figures have congruent angles and proportional sides

AA; **SSS**; **SAS** Similarity Thm

-Corresponding sides of similar triangles are in proportion

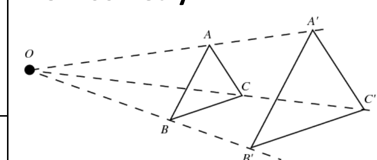
-In a proportion, the product of the means = the product of the extremes

Congruence Triangle Theorems

SAS; AAS; ASA; SSS; HL ≅ Thm

CPCTC (CPCFC)

Dilation – NOT rigid motion, NOT isometry



$D_k(x, y) = (kx, ky)$

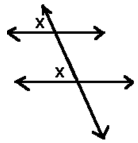
Dilations create similar figures.

A composition of 2 reflections over 2 parallel lines is equivalent to a translation.

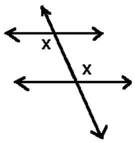
A composition of 2 reflections over 2 intersecting lines is equivalent to a rotation.

Parallel Lines cut by a transversal

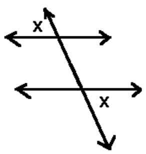
Corresponding Angles are ≅



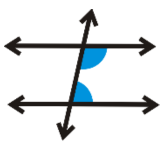
Alternate Interior angles are ≅



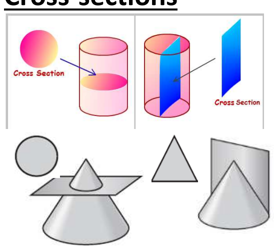
Alternate exterior angles are ≅



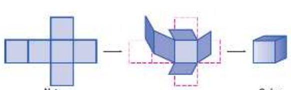
Interior angle on the same side are supplementary



Cross-sections



Nets



Coordinate geometry proofs

Slope formula:

- Parallel lines
- Perpendicular lines
- Right angle

Midpoint Formula:

- Segments bisect each other

Distance Formula:

- Length

Isosceles Triangle

2 congruent sides & 2 congruent base angles

The altitude from the vertex is also the median and angle bisector.

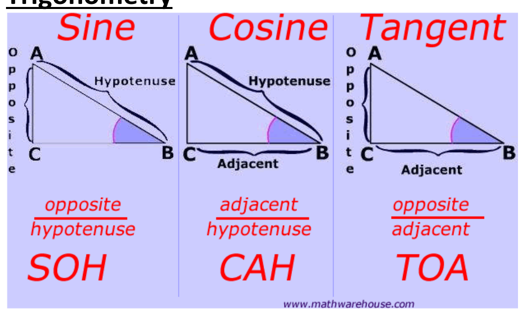
Triangle Inequality

- The sum of the lengths of any 2 sides must be greater than the 3rd
- The longest side is opposite the largest angle
- The measure of the exterior angle is greater than either non-adjacent interior angle

Side Splitter Theorem

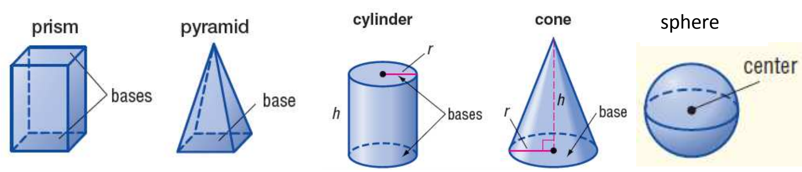
A line drawn parallel to any of the sides of a triangles divides the other 2 sides proportionally

Trigonometry

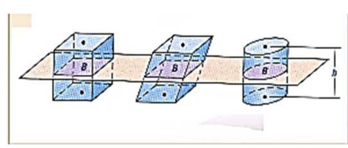


When solving for an angle used the inverse (2nd Key)

3D Solids



Cavalieri's Principle – If 2 solids have the same height and the same cross-sectional area at every level, they have the same volume

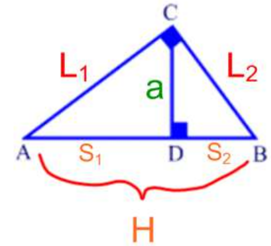


Mass = (Volume) (Density)

Geometric Mean Theorems

Altitude Theorem – The altitude is the geometric mean between the 2 segments of the hypotenuse. $\frac{S1}{a} = \frac{a}{S2}$

Leg Theorem – The leg is the geometric mean between the segment it touches and the whole hypotenuse $\frac{S1}{L1} = \frac{L1}{H}$ and $\frac{S2}{L2} = \frac{L2}{H}$

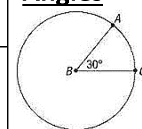


Circles

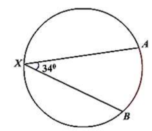
General Form Equation of a Circle
$x^2 + y^2 + Ax + By + C = 0$

Standard center radius form
$(x - h)^2 + (y - k)^2 = r^2$

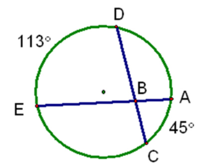
Angles



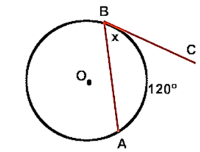
Central angle = arc



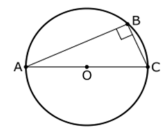
Inscribed angle = ½ arc



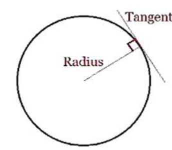
2 chords with vertical angles = $\frac{arc1+arc2}{2}$



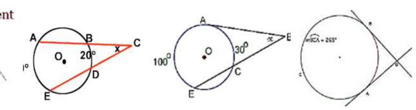
Tangent-chord angle = ½ arc



Angle inscribed in semicircle =

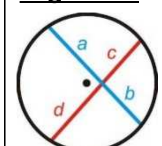


Tangent-radius angle = 90°

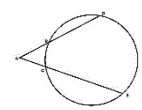


Angle outside circle formed by tangents and secants = $\frac{arc1-arc2}{2}$

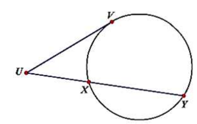
Segments



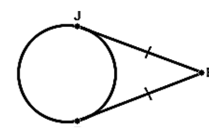
ab=cd



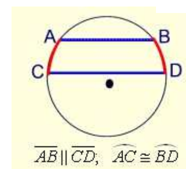
(Outside)(Whole) = (outside)(Whole)

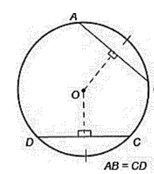


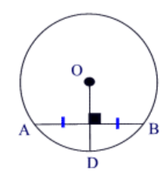
(Outside)(Whole) = (tangent)²

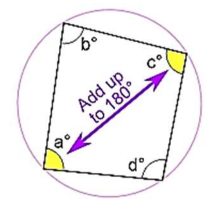


Tangents ≅



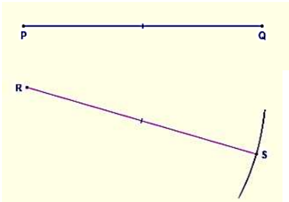




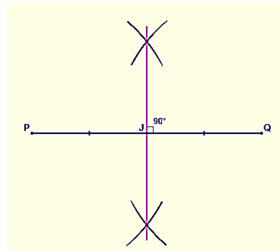


Basic Constructions

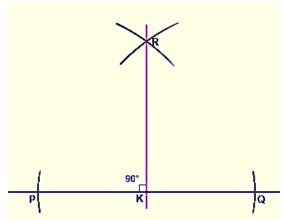
Copy a line segment



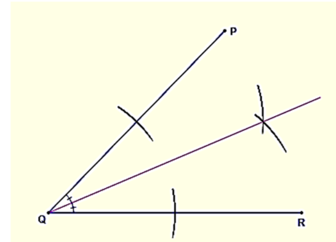
Perpendicular Bisector



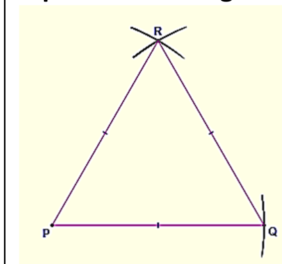
Perpendicular Line thru a point on a line



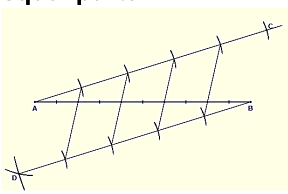
Bisect an angle



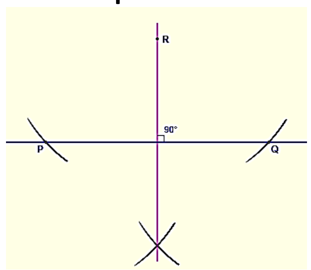
Equilateral Triangle



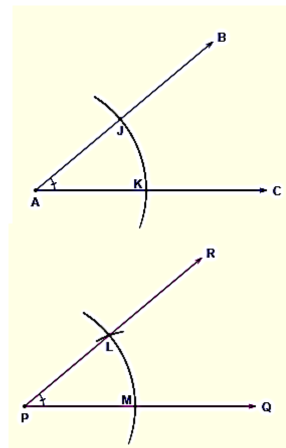
Dividing a segment into equal parts



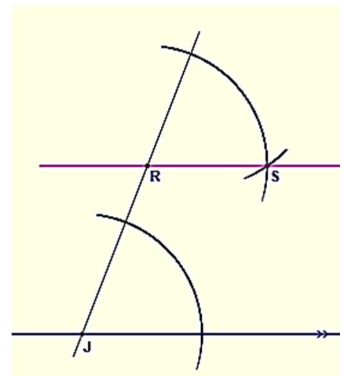
Perpendicular line thru an external point



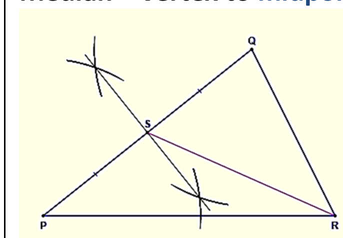
Copy an angle



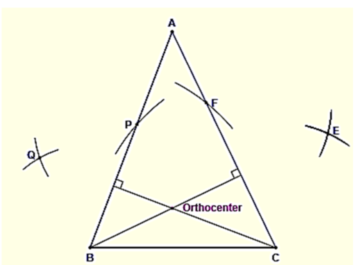
Parallel line



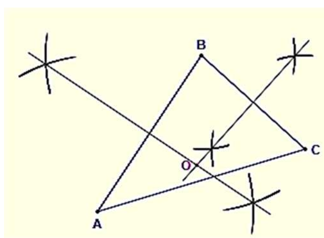
Median – vertex to midpoint



Orthocenter - altitudes

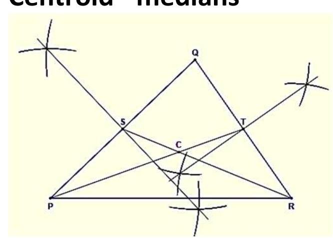


Circumcenter – perpendicular bisectors



- Equidistant to each vertex of the triangle
- Used to circumscribe a circle

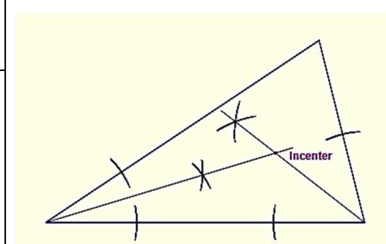
Centroid - medians



- 2:1 ratio

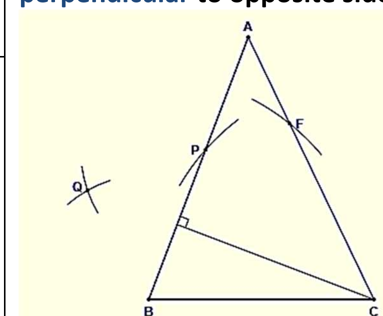
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incenter – angle bisectors

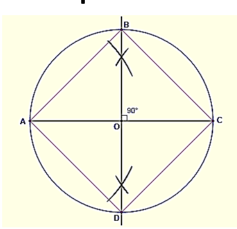


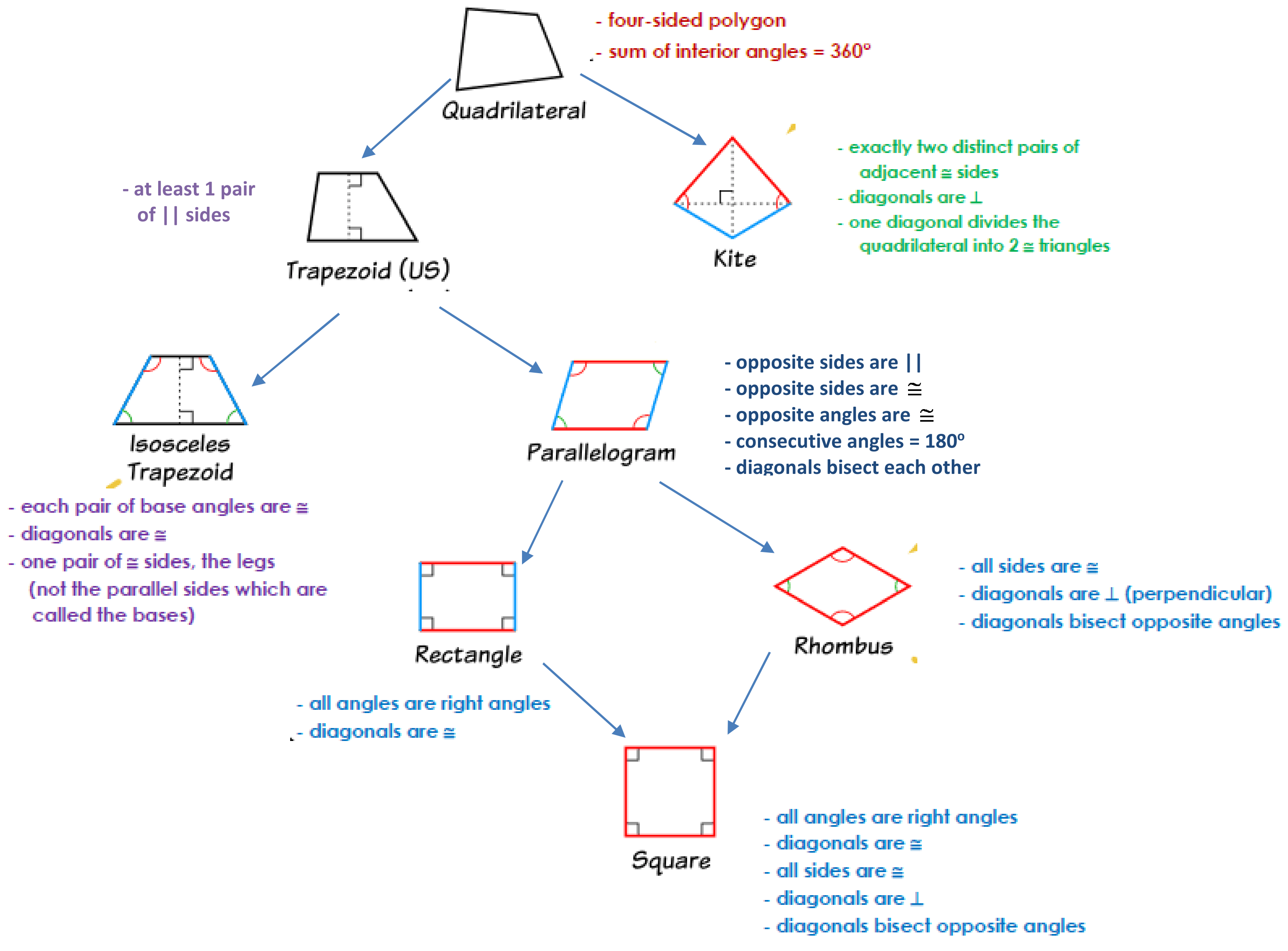
- equidistant to each side of the triangle
- Used to inscribe a circle

Altitude – vertex perpendicular to opposite side



Inscribed square in a circle





Coordinate Proofs:

Trapezoid:
- 1 pair of || sides

Isosceles Trapezoid:
Trapezoid +
- 2 ≅ sides OR
- 2 ≅ diagonals OR

Parallelogram: (pick 1)
- 2 pairs of || sides
- 2 pairs of ≅ sides
- Diagonals bisect each other
- 1 pair of || & ≅ sides

Rectangle:
- 4 right angles OR

Parallelogram +
- 1 right angle OR
- ≅ diagonals

Rhombus:
- 4 ≅ sides OR

Parallelogram +
- 2 consecutive ≅ sides OR
- ⊥ diagonals

Square:
- 4 right ∠'s and 2 consecutive ≅ sides OR
- 4 ≅ sides and 1 right ∠

Kite:
- 2 distinct pairs of ≅ sides