AP® CALCULUS AB/CALCULUS BC
2017 SCORING GUIDELINES

Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

3 : { 1 : uses initial condition; 1 : $f(-6)$; 1 : $f(5)$ }

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

2 : { 1 : considers $x = 2$; 1 : answer with justification }

(d) $f''(-5) = \dfrac{2 - 0}{-6 - (-2)} = -\dfrac{1}{2}$

$\lim_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} = 2$ and $\lim_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3} = -1$

$f''(3)$ does not exist because
$\lim_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} \ne \lim_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3}.$

2 : { 1 : $f''(-5)$; 1 : $f''(3)$ does not exist, with explanation }