Question 4

(a) $H'(6) \approx \dfrac{H(7) - H(5)}{7 - 5} = \dfrac{11 - 6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\dfrac{H(5) - H(3)}{5 - 3} = \dfrac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5,$ H is continuous on $3 \le t \le 5.$

By the Mean Value Theorem, there exists a value $c,$ $3 < c < 5,$ such that $H'(c) = 2.$

$2 : \begin{cases} 1 : \dfrac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\dfrac{1}{10 - 2}\displaystyle\int_2^{10} H(t)\,dt.$

$$\frac{1}{8}\int_2^{10} H(t)\,dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2}\cdot 1 + \frac{2 + 6}{2}\cdot 2 + \frac{6 + 11}{2}\cdot 2 + \frac{11 + 15}{2}\cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\dfrac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x))\cdot\frac{dx}{dt} = \frac{(1 + x)100 - 100x\cdot 1}{(1 + x)^2}\cdot\frac{dx}{dt} = \frac{100}{(1 + x)^2}\cdot\frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2}\cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\dfrac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \dfrac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule