AP® CALCULUS AB 2016 SCORING GUIDELINES

Question 2

For $t \ge 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by

 $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position x = 2 at time t = 4.

- (a) At time t = 4, is the particle speeding up or slowing down?
- (b) Find all times t in the interval 0 < t < 3 when the particle changes direction. Justify your answer.
- (c) Find the position of the particle at time t = 0.
- (d) Find the total distance the particle travels from time t = 0 to time t = 3.

(a)
$$v(4) = 2.978716 > 0$$

 $v'(4) = -1.164000 < 0$

2: conclusion with reason

The particle is slowing down since the velocity and acceleration have different signs.

(b)
$$v(t) = 0 \implies t = 2.707468$$

 $2: \begin{cases} 1: t = 2.707 \\ 1: \text{ justification} \end{cases}$

v(t) changes from positive to negative at t = 2.707. Therefore, the particle changes direction at this time.

(c)
$$x(0) = x(4) + \int_{4}^{0} v(t) dt$$

= 2 + (-5.815027) = -3.815

 $3: \begin{cases} 1 : integral \\ 1 : uses initial condition \\ 1 : answer \end{cases}$

(d) Distance =
$$\int_0^3 |v(t)| dt = 5.301$$

 $2: \begin{cases} 1 : integral \\ 1 : answer \end{cases}$