Potato Differential Equation free response
AP Calculus

Give yourself 15 minutes to do this problem. Practice showing clear, concise work.

At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal

temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato

is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be

modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{4}(H - 27)$, where $H(t)$ is

measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the
internal temperature of the potato at time $t = 3$.

(b) Use $\dfrac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the

internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function

G that satisfies the differential equation $\dfrac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius

and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the

potato at time $t = 3$?