Free response with Squeeze Theorem from 2019 #6

(a) $h'(2) = \frac{2}{3}$	1 : answer
(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$ $a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$	3 : { 1 : form of product rule 1 : $a'(x)$ 1 : $a'(2)$
(c) Because h is differentiable, h is continuous, so $\lim_{x \to 2} h(x) = h(2) = 4$. Also, $\lim_{x \to 2} h(x) = \lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$. Because $\lim_{x \to 2} \left(x^2 - 4\right) = 0$, we must also have $\lim_{x \to 2} \left(1 - (f(x))^3\right) = 0$. Thus $\lim_{x \to 2} f(x) = 1$. Because f is differentiable, f is continuous, so $f(2) = \lim_{x \to 2} f(x) = 1$. Also, because f is twice differentiable, f' is continuous, so $\lim_{x \to 2} f'(x) = f'(2)$ exists. Using L'Hospital's Rule, $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \to 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4$. Thus $f'(2) = -\frac{1}{3}$.	4 : { 1 : $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$ 1 : $f(2)$ 1 : L'Hospital's Rule 1 : $f'(2)$
(d) Because g and h are differentiable, g and h are continuous, so $\lim_{x \to 2} g(x) = g(2) = 4$ and $\lim_{x \to 2} h(x) = h(2) = 4$. Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \to 2} k(x) = 4$. Also, $4 = g(2) \le k(2) \le h(2) = 4$, so $k(2) = 4$. Thus k is continuous at $x = 2$.	1 : continuous with justification