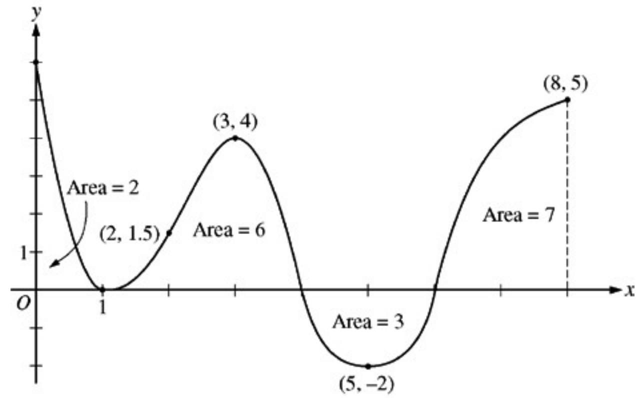
AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.	1 : answer with justification
(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint. $f(0) = f(8) + \int_8^0 f'(x)\,dx = f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$; $f(6) = f(8) + \int_8^6 f'(x)\,dx = f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$; $f(8) = 4$. The absolute minimum value of f on the closed interval $[0, 8]$ is -8.	3 : { 1 : considers $x = 0$ and $x = 6$; 1 : answer; 1 : justification }
(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.	2 : { 1 : answer; 1 : explanation }
(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$; $g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$	3 : { 2 : $g'(x)$; 1 : answer }