Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim_{x\to 0^-} (1 - 2\sin x) = 1$ $\lim_{x\to 0^+} e^{-4x} = 1$ $f(0) = 1$ So, $\lim_{x\to 0} f(x) = f(0)$. Therefore f is continuous at $x = 0$.	2 : analysis
(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$ $-2\cos x \neq -3$ for all values of $x < 0$. $-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$. Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.	$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$