Separable Differential Equations with linear approximation 2013 #6

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = e^0(3\cdot 1^2 - 6\cdot 1) = -3$

An equation for the tangent line is $y = -3(x - 1)$.

$f(1.2) \approx -3(1.2 - 1) = -0.6$

$3: \begin{cases} 1: \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1: \text{tangent line equation} \\ 1: \text{approximation} \end{cases}$

(b) $\frac{dy}{e^y} = (3x^2 - 6x)\,dx$

$\int \frac{dy}{e^y} = \int (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

$y = -\ln(-x^3 + 3x^2 - 1)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$6: \begin{cases} 1: \text{separation of variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables