AP® CALCULUS AB
2014 SCORING GUIDELINES

Question 6

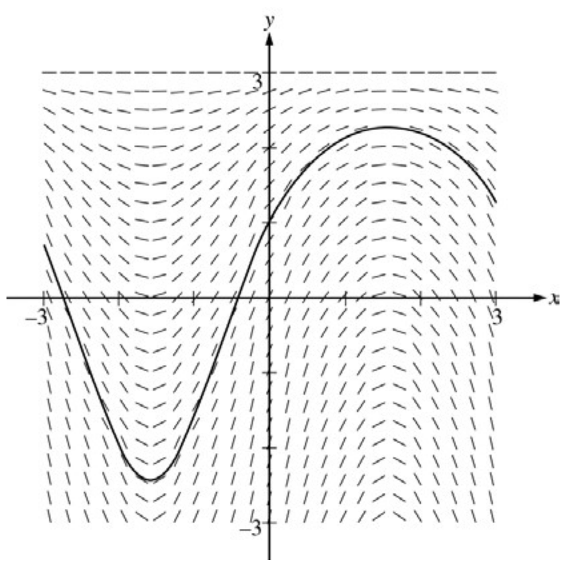
Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a)



1 : solution curve

(b) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(0,\, 1)} = 2\cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$f(0.2) \approx 2(0.2) + 1 = 1.4$

2 : { 1 : tangent line equation; 1 : approximation }

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$\int \frac{dy}{3 - y} = \int \cos x \, dx$

$-\ln|3 - y| = \sin x + C$

$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$

$-\ln|3 - y| = \sin x - \ln 2$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$3 - y = 2e^{-\sin x}$

$y = 3 - 2e^{-\sin x}$

Note: this solution is valid for all real numbers.

6 : { 1 : separation of variables; 2 : antiderivatives; 1 : constant of integration; 1 : uses initial condition; 1 : solves for y }

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables