DUSO MATHEMATICS LEAGUE
INDIVIDUAL QUESTIONS - MEET #1
OCTOBER 27, 2010

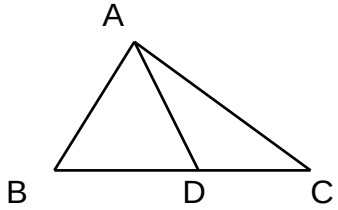
1. ALG. 1 (MATH A) 6 MINUTES

"DUSOML" is an acronym for "Dutchess Ulster Sullivan Orange Math League". The characters in "DUSOML" and the characters in "2010-2011" are cycled separately as shown below and placed in a vertical list. When the list is consecutively numbered, the seventh occurrence of "DUSOML 2010-2011" will appear on line N. Determine the number N.

1.	DUSOML 2010-2011
2	LDUSOM 12010-201
3	MLDUSO 112010-20
4	OMLDUS 0112010-2
5	SOMLDU 20112010-
6	USOMLD -20112010
7	DUSOML 0-2011201
	. . .
N.	DUSOML 2010-2011

2. GEOMETRY (MATH B) 5 MINUTES

In ΔABC, point D lies on $\overline{BC}$ such that BD :DC = 3 : 2. If the area of ΔABC is $40\sqrt{2}$ find the area of ΔABD in simplest radical form.



3. ALG. 2 / TRIG (MATH B) 5 MINUTES

Find *all* the real roots of the following equation.

$$\left|x^3\right| - 3x^2 - 4|x| + 12 = 0$$

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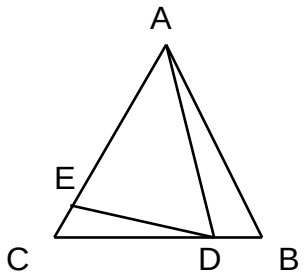
4. GEOMETRY (MATH A) 6 MINUTES

Given ΔABC with $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AE}$, and $m\angle BAD = 30°$.
Find the number of degrees in $m\angle CDE$.



5. ALG 1 (MATH A) 5 MINUTES

Lois and Jimmy each randomly and independently select a whole number from 1 to 25, inclusive. It is possible that they each select the same number. What is the probability, ***in simplest form*** that Jimmy's number is greater than Lois' number?

6. ALG. 2 / TRIG (MATH B) 5 MINUTES

The functions *f* and *g* are functions whose domain and range are real numbers, with $g(x) = 2x-1$ and $f(g(x)) = 10x - 4$.

Find the value of $f(2)$ and express it in simplest form.

DUSO MATHEMATICS LEAGUE
RELAY TEAM QUESTION- MEET #1
OCTOBER 27, 2010

1. If $a+b=79$, where *a* is at least six units larger than *b*, and *b* is a prime number, what is the greatest prime number ***b*** could be?

2. A woman flew from Billings, Montana to Fort Lauderdale, Florida , and then back to Billings again. Because her arms got tired, the flight to back to Billings took 40% longer than the flight to Florida. If the round trip took 12 hours, how many hours did the flight to Billings take? Add your answer to **TNYWR.**

3. The distance between (3, 5) and (***k***, 3) is $\sqrt{40}$ units. If ***k*** is positive, find ***k*** and add this to **TNYWR**.

.

4. Find $\sqrt[3]{4.8}$ to the nearest tenth. (Hint: Guess and check.)
Now multiply your result by 10, and then add that number to **TNYWR**.

5. Beyonce and Shakira start from the same point at the same time. Beyonce travels due east at 6 m.p.h. Shakira travels due south at 7 m.p.h. To the nearest whole number of miles, what is the shortest distance between them after 3 hours?
Add this to **TNYWR**.

DUSO MATHEMATICS LEAGUE
SOLUTIONS - SOLUTIONS - MEET #1
OCTOBER 27, 2010

Answers for RELAY TEAM QUESTION

1). 31 **2).** 38 **3).** 47 **4).** 64 **5).** 92

(Solutions for the Relay question)

1).). Let $b = 79 - a$, so $a - (79 - a) \geq 6$

Solving we get $2a \geq 85$, $a \geq 42.5$ which means $b \leq 36.5$ and since b is a prime number, the largest prime must be 31. Thus a would be 48, which is at least 6 units larger than 31.

So b= 31

3). (3, 5) and (***k***, 3)

$$\sqrt{(k-3)^2 + (3-5)^2} = \sqrt{40}$$
$$(k-3)^2 + (-2)^2 = 40$$
$$(k-3)^2 = 36$$
$$(k-3)^2 = 36$$
$$k - 3 = 6 \quad \text{or} \quad k - 3 = -6$$
k = 9 or k = -3 (reject)

So 9 + TNYWR = 9 + 38= 47

2). x = time to Florida
1.4x = time to Billings

$$x + 1.4x = 12$$
$$2.4x = 12$$
$$x = 5$$
$$1.4x = 7$$

7 hours to Billings

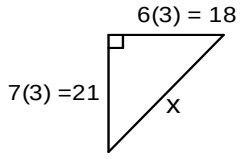
So 7 + TNYWR = 7 + 31 = 38

4) $(1.5)^3 = 3.375$
$(1.6)^3 = 4.096$
$(1.7)^3 = 4.913$

So $\sqrt[3]{4.8}$ to the nearest tenth is 1.7.
Now 10(1.7) = 17

So 17 + TNYWR = 17 + 47 = 64

5)



$$18^2 + 21^2 = x^2$$
$$324 + 441 = x^2$$
$$x^2 = 765, \quad x = \sqrt{765}$$
$$x = 28 \text{ (nearest whole)}$$

$27^2 = 729, 28^2 = 784,$

So 28 + TNYWR = 28 + 64 = 92

Individual Questions selected from the DUSO Question Bank, revised/editted by J.S.
Relay team question written for DUSO by J.A. DUSO Editor J.S.

DUSO MATHEMATICS LEAGUE
SOLUTIONS - MEET #1
OCTOBER 27, 2010

Answers for INDIVIDUAL QUESTIONS

1). 109 **2).** $24\sqrt{2}$ **3).** -3,-2,2,3 **4).** 15 or 15º **5).** $12/25$ or .48 **6).** 11

Solutions for the Individual questions)

1). There are 6 characters in "DUSOML" and 9 characters in "2010-2011". Since the LCM of 6 and 9 is 18, ***both*** sets of characters have completed cycles every 18 lines. So "DUSOML 2010-2011" appears on lines 1, 19, 31, …etc.

Thus the 7th occurrence of "DUSOML 2010-2011" will appear on line N = 18(6) +1 = 109

3). Case I : $x \geq 0$

$x^3 - 3x^2 - 4x + 12 = 0$

$x^2(x-3) - 4(x-3) = 0$

$(x-3)(x^2-4) = 0$ so $(x-3)(x-2)(x+2) = 0$

Thus $x = 3$, $x = 2$, $x = -2$

Case II : $x < 0$

$(-x)^3 - 3(-x)^2 - 4(-x) + 12 = 0$

$-x^3 - 3x^2 + 4x + 12 = 0$

$-x^2(x+3) + 4(x+3) = 0$

$(x+3)(4-x^2) = 0$ so $(x+3)(2-x)(2+x) = 0$

Thus $x = -3$, $x = 2$, $x = -2$

{-3, -2, 2, 3}

5). There are 25 x 25 = 625 possibilities. Of these, 25 times they select the same number (1,1 or 2,2 or 3,3 ….). That means they select different numbers in 625 -25 = 600 cases. In half of these cases, or in 300 cases Jimmy's number will be greater.

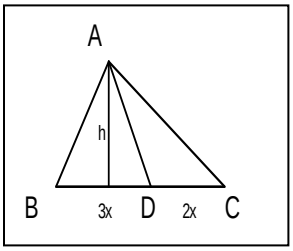
So

P(Jimmy has the greater number) = $\frac{300}{625}$

However, since the question requires that our probability answer be in its simplest form, we must simplify $\frac{300}{625}$ to $\frac{12}{25}$ or .48

2) Let BD =3x, DC = 2x

Thus BC = 5x



Area Δ ABC = $40\sqrt{2}$

So $\frac{1}{2}(5x)h = 40\sqrt{2}$

$xh = 16\sqrt{2}$

Area Δ ABD =

$\frac{1}{2}(3x)h = \frac{3}{2}xh = \frac{3}{2}(16\sqrt{2}) = 24\sqrt{2}$

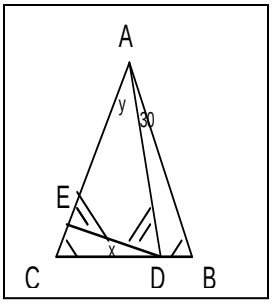
OR

Since the triangles have the same height, their areas will be in the ratio of 3 : 5. $\frac{3}{5}(40\sqrt{2}) = 24\sqrt{2}$

4). Let m<CDE = x, m<CAD = y

Now m<ACB = m<ABC =

$\frac{1}{2}(180 - (y + 30)) = 75 - \frac{1}{2}y$



And m<AED = m<ADE =

$\frac{1}{2}(180 - y) = 90 - \frac{1}{2}y$

Using an exterior angle relationship

m<AED = m<ECD + m<EDC

(Note: <ECD is the same as <ACB)

$90 - \frac{1}{2}y = 75 - \frac{1}{2}y + x$ thus $x = 15$

6) $f(2) = f(g(x))$ when $g(x) = 2$

So for $g(x) = 2 = 2x-1$

$2x = 3$

$x = \frac{3}{2}$

Since $g(x) = 2$ when $x = \frac{3}{2}$

We use = $f(g(x))$ = 10x -4 with $x = \frac{3}{2}$

$f(2) = 10\left(\frac{3}{2}\right) - 4 = 11$