DUSO Mathematics League 2011 - 2012

Contest #1. **Calculators are not permitted on this contest.**

Part I. **ALGEBRA I** *Time Limit: 8 minutes*

The word "compute" calls for an exact answer in simplest form.

1 - 1. Maggie the mathematician is running a 5000-meter race. At some point during the race, she realizes that the distance she has yet to run is $\frac{3}{5}$ of the distance she has already run. Compute the number of meters she has yet to run.

1 - 2. Solve: $(3x^2 + 9x + 2)(x - 1) = (3x^2 + x - 1)(x + 2)$

DUSO Mathematics League 2011 - 2012

Contest #1. **Calculators are not permitted on this contest.**

Part II. **GEOMETRY** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

1 - 3. Consider the system of inequalities below.

$$\begin{aligned} x &\geq -1 \\ 4 - x &\geq -y \\ 4x + y &= 6 \end{aligned}$$

The system's solution is a segment of length d. Compute d.

1 - 4. Chords AB and CD of circle O cross at point E. Suppose $AB = 6$ cm, and the lengths of AE, BE, CE, and DE are integers. Compute the possible lengths of CD, in cm.

DUSO Mathematics League 2011 - 2012

Contest #1. **Calculators are not permitted on this contest.**

Part III. **ALGEBRA II / ADVANCED TOPICS** *Time Limit: 12 minutes*

The word "compute" calls for an exact answer in simplest form.

1 - 5. Let $\binom{n}{r}$ denote the number of ways to choose r objects from a set of n objects.

If $\dfrac{\binom{n}{3}}{\binom{n-2}{2}} = 4$, compute both values of n.

1 - 6. Given that the roots of $x^3 - 19x^2 + bx - 216 = 0$ are positive and in geometric progression, compute b.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

R-1. Jimmy subtracts 4 from a magic number, multiplies the result by 5, and adds 6 to that result. If his final answer is 126, compute the magic number.

R-2. *Let N be the number you will receive.* The x-intercept of the line $2x + 7y = N$ is A. The y-intercept of the line $2x + 7y = N$ is B. Compute $A + B$.

R-3. *Let N be the number you will receive.* The numbers X, Y, and Z are such that $7X - 8Y = 24$ and $15Y + 7Z = N$. Compute the mean (average) of X, Y, and Z.

R-4. *Let N be the number you will receive.* Juan and Maria go to the candy store. Juan buys 4 Tootsy Rolls and 5 Bazuka Joes for $\$N$. Maria buys 5 Tootsy Rolls and 3 Bazuka Joes for \$1.85. Compute the cost of a Bazuka Joe in cents.

R-5. *Let N be the number you will receive.* The line $y = \frac{2}{3}x - N$ passes through many points in the fourth quadrant, but only some of those have integer coordinates. Compute the number of points in the fourth quadrant on the graph of $y = \frac{2}{3}x - N$ that have integer coordinates.