DUSO Mathematics League 2011 - 2012

CONTEST #1.

SOLUTIONS

1 - 1. Let the distance Maggie has already run be x. Then the distance she has yet to run is $\frac{3x}{5}$, and the total distance she will run is $\frac{8x}{5} = 5000$, so $x = 3125$ and $\frac{3x}{5} = \frac{3}{5} \cdot 3125 = \mathbf{1875}$.

1 - 2. We expand each side to obtain $3x^3 + 6x^2 - 7x - 2 = 3x^3 + 7x^2 + x - 2$, which simplifies to $x^2 + 8x = 0$, so our solutions are $\{\mathbf{0}, \mathbf{-8}\}$.

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1 - 3. The line $4x + y = 6$ intersects the line $x = -1$ at $(-1, 10)$ and the line $4 - x = -y$ at $(2, -2)$. We desire the length between those two intersection points, which is $\sqrt{(2+1)^2 + (-2-10)^2} = \sqrt{153} = \mathbf{3\sqrt{17}}$.

1 - 4. We use the Power of a Point Theorem to compute possible lengths for CE and DE. In every case, we have $(AE)(BE) = (CE)(DE)$. If $AE = 1$ and $BE = 5$, then the only possible lengths for CE and DE are 1 and 5, so $CD = 6$. If $AE = 2$ and $BE = 4$, then the only possible lengths for CE and DE are 2 and 4 or 1 and 8, so $CD = 6$ or $CD = 9$. If $AE = 3$ and $BE = 3$, then the only possible lengths for CE and DE are 1 and 9 or 3 and 3, so $CD = 10$ or $CD = 6$. Thus, the possible lengths are **6, 9, or 10**.

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1 - 5. The equation may be rewritten as $\dfrac{\dfrac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1}}{\dfrac{(n-2)(n-3)}{2 \cdot 1}} = \frac{n(n-1)}{3(n-3)} = 4$, which can be solved to obtain $n = \{\mathbf{4}, \mathbf{9}\}$.

1 - 6. The roots are p, pr, and pr^2. The product of the roots is $p^3r^3 = 216$, so $pr = 6$. The sum of the roots is 19, so $p + pr + pr^2 = p(1 + r + r^2) = 19$. The value of b is the sum of the pairwise products of the roots, so
$b = p^2r + p^2r^2 + p^2r^3 = p^2r(1 + r + r^2) = (pr)(p(1 + r + r^2)) = 6 \cdot 19 = \mathbf{114}$.

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R-1. Jimmy subtracts 4 from a magic number, multiplies the result by 5, and adds 6 to that result. If his final answer is 126, compute the magic number.
R-1Sol. We solve $5(x-4)+6=126$. $5(x-4)+6=126 \rightarrow 5(x-4)=120 \rightarrow x-4=24$, so $x=\mathbf{28}$.

R-2. *Let N be the number you will receive.* The x-intercept of the line $2x+7y=N$ is A. The y-intercept of the line $2x+7y=N$ is B. Compute $A+B$.
R-2Sol. The x-intercept A of the line is found by solving $2A=N \rightarrow A=N/2$. The y-intercept B is found by solving $7B=N \rightarrow B=N/7$. We have $A+B=\frac{N}{2}+\frac{N}{7}=\frac{9N}{14}$. Substituting, $A+B=\frac{9\cdot 28}{14}=\mathbf{18}$.

R-3. *Let N be the number you will receive.* The numbers X, Y, and Z are such that $7X-8Y=24$ and $15Y+7Z=N$. Compute the mean (average) of X, Y, and Z.
R-3Sol. Add the two equations together. $7X+7Y+7Z=24+N$, so $X+Y+Z=\frac{24+N}{7}$, so the average of the three numbers is $\frac{24+N}{21}$. Substituting, we have $\frac{24+18}{21}=\mathbf{2}$.

R-4. *Let N be the number you will receive.* Juan and Maria go to the candy store. Juan buys 4 Tootsy Rolls and 5 Bazuka Joes for \$$N$. Maria buys 5 Tootsy Rolls and 3 Bazuka Joes for \$1.85. Compute the cost of a Bazuka Joe in cents.
3-4Sol. We solve the system $4T+5B=100N$ and $5T+3B=185$. Multiplying the first equation by 5 and the second by 4 yields $20T+25B=500N$ and $20T+12B=740$. Subtracting, we have $13B=500N=740$, or $B=\frac{500N-740}{13}$. Substituting, we have $B=\frac{260}{13}=\mathbf{20}$.

R-5. *Let N be the number you will receive.* The line $y=\frac{2}{3}x-N$ passes through many points in the fourth quadrant, but only some of those have integer coordinates. Compute the number of points in the fourth quadrant on the graph of $y=\frac{2}{3}x-N$ that have integer coordinates.
R-5Sol. From the y-intercept of -20, the line goes up 2 units for every 3 it goes across. There won't be an integer x-coordinate when $y=-19$, but there will be at $y=-18$, and there will be at every even integer y-coordinate between -18 and -2. The number of such points is $\mathbf{9}$.