DUSO Mathematics League 2012 - 2013

CONTEST #2.

SOLUTIONS

2 - 1. $\boxed{38}$ We solve the equation $5N + 10(85 - N) = 660$ to obtain $-5N = -190$, so $N = \mathbf{38}$.

2 - 2. $\boxed{12}$ Let x represent Sage's age. Then $3x$ represents her father's age and $3x - 3$ represents her mother's age. When she was born, her parents were $2x$ and $2x - 3$ years old. Thus, we solve $4x - 3 = 45$ to obtain $x = \mathbf{12}$.

---

2 - 3. $\boxed{6}$ We solve $s^3 = 6s^2$ to obtain $s = 6$.

2 - 4. $\boxed{2\sqrt{34}}$ If we drop an altitude from C to $\overline{AB}$, we see that $\sin 60° = \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{BC} \Rightarrow BC = 6$. Imagine a line through C parallel to the two parallel lines and check out pairs of same-side interior angles... $m\angle BCD = 60° + 30° = 90°$, so $BD = \sqrt{6^2 + 10^2} = \sqrt{136} = \mathbf{2\sqrt{34}}$.

---

2 - 5. $\boxed{\frac{1}{5}}$ There are $\frac{6!}{3!} = 120$ ways to arrange the six letters. Treating the three E's as a mega-letter, there are $4! = 24$ ways to keep the three E's together. Thus, our probability is $\frac{24}{120} = \mathbf{\frac{1}{5}}$.

2 - 6. $\boxed{-\frac{1}{2} < x < \frac{1}{2}}$ We note that if $x \geq \frac{1}{2}$, then the expression on the left side of the inequality is $4x$. If $-\frac{1}{2} < x < \frac{1}{2}$, the expression on the left is equal to $1 - 2x + 2x + 1 = 2$, which is greater than $|4x|$ on the interval. If $x < -\frac{1}{2}$, the expression on the left is $(1 - 2x) + (-2x - 1) = -4x$, whose absolute value is the same as $|4x|$ for those values of x. Thus, our interval of solution is $\mathbf{-\frac{1}{2} < x < \frac{1}{2}}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

T-1. For two acute angles A and B (measured in degrees), we have $\sin A + \sin B = \dfrac{\sqrt{8}+\sqrt{12}}{4}$ and $\sin A \cdot \sin B = \dfrac{\sqrt{96}}{16}$. If $A < B$, compute (A, B).

T-1Sol. $\boxed{(45, 60)}$ The fractions we are adding or multiplying are $\frac{\sqrt{2}}{2}$ and $\frac{\sqrt{3}}{2}$, so our angles are $45°$ and $60°$.

T-2. Adam and Beth, working together, paint $\frac{2}{3}$ of a wall. Carly, who could paint the whole wall by herself in 8 hours, joins Adam and Beth, and they three together finish painting the wall in 2 hours. Compute the total number of hours Beth spent painting the wall.

T-2Sol. $\boxed{\mathbf{18}}$ Let x be the number of hours Beth spent painting the first $2/3$ of the wall, and A and B be the rates at which Adam and Beth paint. We have $\frac{x}{A} + \frac{x}{B} = \frac{2}{3} \to x\left(\frac{1}{A} + \frac{1}{B}\right) = \frac{2}{3}$. We also have $\frac{2}{A} + \frac{2}{B} + \frac{2}{8} = \frac{1}{3}$ from the information about the other third of the wall. This second equation tells us that $\frac{2}{A} + \frac{2}{B} = \frac{1}{12} \to \frac{1}{A} + \frac{1}{B} = \frac{1}{24}$. Substituting into the first equation, we have $x \cdot \frac{1}{24} = \frac{2}{3} \to x = 16$. Our answer is $16 + 2 = \mathbf{18}$ hours.

T-3. Compute the sum of the infinite series: $\frac{1}{3} + \frac{4}{9} + \frac{7}{27} + \frac{10}{81} + \frac{13}{243} + \cdots$

T-3Sol. $\boxed{\mathbf{\frac{5}{4}}}$ Let the sum be represented by S. $S = \frac{1}{3} + \frac{4}{9} + \frac{7}{27} + \frac{10}{81} + \frac{13}{243} + \cdots$, so $\frac{S}{3} = \frac{1}{9} + \frac{4}{27} + \frac{7}{81} + \frac{10}{243} + \frac{13}{729} + \cdots$. Subtracting, we have $\frac{2S}{3} = \frac{1}{3} + \frac{3}{9} + \frac{3}{27} + \frac{3}{81} + \cdots$. Utilizing the formula for the sum of an infinite geometric series, we have $\frac{2S}{3} = \frac{1}{3} + \dfrac{\frac{1}{3}}{1-\frac{1}{3}}$, so $\frac{2S}{3} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$, and thus $S = \mathbf{\frac{5}{4}}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com