Solving Systems of Equations (Word Problems) Two Variables and Two Equations

Directions QUESTIONS 1-4: Follow the steps for solving word problems that you have already been given to solve the following problems. You must write a system of equations and solve algebraically

1) Guy and Jim work at a furniture store. Guy is paid $185 per week plus 3% of his total sales in dollars, \( x \), which can be represented by \( g(x) = 185 + 0.03x \). Jim is paid $275 per week plus 2.5% of his total sales in dollars, \( x \), which can be represented by \( f(x) = 275 + 0.025x \). Determine the value of \( x \), in dollars, that will make their weekly pay the same.

\[
\begin{align*}
185 + 0.030x &= 275 + 0.025x \\
-0.025x &= -0.025x \\
185 + 0.005x &= 275 \\
-185 &= -185 \\
0.005x &= 90 \\
\frac{0.005x}{0.005} &= \frac{90}{0.005} \\
x &= 18,000
\end{align*}
\]

If the sell $18,000, their weekly pay will be the same.

2) During the 2010 season, football player McGee’s earnings, \( m \), were 0.005 million dollars more than those of his teammate Fitzpatrick’s earnings, \( f \). The two players earned a total of 3.95 million dollars. Determine the amount each player earned, in millions of dollars?

\[
\begin{align*}
m &= 0.005 + f \\
m + f &= 3.95
\end{align*}
\]

\[
\begin{align*}
m + f &= 3.95 \\
(0.005 + f) + f &= 3.95 \\
0.005 + 2f &= 3.950 \\
-0.005 &= -0.005 \\
2f &= 3.945 \\
\frac{2f}{2} &= \frac{3.945}{2} \\
f &= 1.9725
\end{align*}
\]

McGee earnings are $1,977,500.

Fitzpatrick’s earnings are $1,972,500.
3) Last week, a candle store received $355.60 for selling 20 candles. Small candles sell for $10.98 and large candles sell for $27.98. How many large candles did the store sell?

Let \( l \) = number of large candles
\( s \) = number of small candles

\[
\begin{align*}
\quad l + s &= 20 \\
10.98s + 27.98l &= 355.60 \\
10.98(20 - l) + 27.98l &= 355.60 \\
21.960 - 10.98l + 27.98l &= 355.60 \\
21.960 + 17l &= 355.60 \\
17l &= 333.64 \\
l &= \frac{333.64}{17} \\
l &= 19.6 \\
s &= 20 - l \\
s &= 20 - 19.6 \\
s &= 0.4
\end{align*}
\]

The store sold 19 large candles.

4) Jacob and Zachary go to the movie theater and purchase refreshments for their friends. Jacob spends a total of $18.25 on two bags of popcorn and three drinks. Zachary spends a total of $27.50 for four bags of popcorn and two drinks. What is the price of one popcorn and the price of one drink (to the nearest cent)?

Let \( p \) = price of one popcorn
\( d \) = price of one drink

Jacob \( \Rightarrow \) \( 2p + 3d = 18.25 \)
Zachary \( \Rightarrow \) \( 4p + 2d = 27.50 \)

\[
\begin{align*}
-2(2p + 3d = 18.25) &\Rightarrow -4p - 6d = 36.50 \\
4p + 2d &= 27.50 \\
4d &= -9.00 \\
\frac{4d}{4} &= \frac{-9.00}{4} \\
d &= 2.25
\end{align*}
\]

One popcorn costs $5.75.

One drink costs $2.25.
5) Albert says that the two systems of equations shown below have the same solutions.

<table>
<thead>
<tr>
<th>First System</th>
<th>Second System</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2(8x + 9y = 48) )</td>
<td>( 8x + 9y = 48 )</td>
</tr>
<tr>
<td>(-2(12x + 5y = 21) )</td>
<td>(-8.5y = -51 )</td>
</tr>
</tbody>
</table>

Determine if you agree by solving each system and showing whether the solutions are the same or not.

\[
\begin{align*}
24x + 27y &= 144 \\
-24x - 10y &= -42 \\
\hline
17y &= 102 \\
\frac{17}{17} y &= \frac{102}{17} \\
y &= 6
\end{align*}
\]

\[
\begin{align*}
-8.5y &= -51 \\
\frac{-8.5}{-8.5} y &= \frac{-51}{-8.5} \\
y &= 6
\end{align*}
\]

\[
\begin{align*}
8x + 9y &= 48 \\
8x + 9(6) &= 48 \\
8x + 54 &= 48 \\
-54 &= -54 \\
\hline
8x &= -6 \\
\frac{8}{8} x &= \frac{-6}{8} \\
x &= \frac{-3}{4}
\end{align*}
\]

\[
\begin{align*}
\text{Solution} \quad \left( \frac{-3}{4}, 6 \right)
\end{align*}
\]

\[
\begin{align*}
\text{Solution} \quad \left( \frac{-3}{4}, 6 \right)
\end{align*}
\]

The solution for both systems is \( \left( \frac{-3}{4}, 6 \right) \).